Delay-Based Feedback Formation Control for Unmanned Aerial Vehicles with Feedforward Components

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Abstract. This paper mainly studies the delay-based feedback formation control problem with feedforward components for multiple unmanned aerial vehicles (UAVs) system. First, the kinematic equation of the leader-follower UAVs formation system with regard to three directions is established, and the communication network topology between UAVs is presented. Second, by intentionally introducing time-delay into feedback control channel, a delay-based feedback formation control scheme with feedforward components is proposed for the multiple UAVs system. The sufficient conditions of asymptotical stability of closed-loop system are derived based on the linear matrix inequality (LMI) theory, and the design method of the delayed formation controller is presented. The effectiveness of this control scheme is verified based on simulation results, which show that under the designed formation controller, the formation performance of the multiple UAVs system can be guaranteed effectively.

Keywords: Multiple agents · UAV · Formation control · Leader-follower · Delay feedback

1 Introduction

Multiple unmanned aerial vehicles (UAVs) system can coordinate and cooperate with each other and use the advantages of swarms to complete tasks. It has been widely applied in practical fields, such as volcano monitoring [1], target detection [2], coverage path planning [3], logistics delivery [4], large-scale rescue search [5],

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and so on. Due to the important roles in the cooperative control of multiple UAVs system, the formation control has become one of the most increasing attractive issues in recent decades. A great deal of results has been achieved with regard to the coordinate control of multiple agent formation problems [7,17,18]. From the structure perspective in the existing representative literatures, the approaches of multiple UAVs formation control mainly focus on leader-follower method [6], behavior-based method [7], virtual structure method [8], artificial potential field method [9] and so on. Relatively speaking, the leader-follower formation is a widely used formation approach.

Note that the real-time communication between UAVs is a significant guarantee of the success of formation process. Nevertheless, time-delay is an unavoidable phenomenon in real system due to time consumption of the signal input and transmission in communication process, which leads to the disturbance cannot be detected early and the control effect cannot be implemented in time. Therefore, analysis and design for the systems with time-delay have always been a problem of great concern in the formation control area. One attractive research is the stability of control systems with time-delay [10–12]. [10] incorporated time-delay into the model and proposed a control strategy composed of an adaptive fuzzy logic controller and a PID controller for stability. In order to achieve the sufficient condition for stable formation feasibility, the method of Linear Matrix Inequalities (LMIs) is used in [11] and a Lyapunov function is introduced in [12] for time-delay system.

There are also plenty of studies on how to reduce the negative impact of time-delay on the control system. To attenuate the influence of state delay of the agents, an observer predictor is proposed in [13] by predicting the future information about states and disturbances, and an adaptive neural network-based backstepping controller with appropriate control signal designed by Lyapunov function is introduced in [14]. The unknown time-delay of the communication network between the object and the controller is considered by proposing a discrete-time adaptive control method in [15], and a time-varying communication delay is studied for a leader-following formation control of second-order nonlinear systems in [16]. To compensate the multiple communication time-delay, a single predictor-feedback scheme is presented in [17] and a model predictive formation controller is designed in [18]. On the other hand, time-delay plays a positive effect for system performance. In [19], one can see that time-delays are deliberately introduced to reduce the vibration of the offshore structures and can improve the performance of systems. With regard to the recent progress of the formation control problem for the UAV systems, one can see [20–23], and the references therein.

Based on the network topology of [23], this paper aims to design a delay-based formation controller with feedforward components for the multiple UAVs system by artificially introduceing time-delay into control channel, and investigates the effects of the timed-delay on the formation performance. The main contribution of this paper is to analyze the sufficient conditions of delayed system stability through Lyapunov-Krasovskii method and to explore the maximum admissible time-delay on precondition of system stable, and also to study the performance of UAVs formation system. On basis of a leader-follower UAVs, a communication network topology is presented first. Then, by introducing time-delays into feedback control channel, a delay-based feedback formation control scheme with feedforward components is proposed. The sufficient condition of asymptotical stability of system is derived, and the design method of the delayed formation controller is developed. Simulation results show that the delay-based feedback formation control scheme with feedforward components is effective to guarantee the formation performance of the multiple UAVs system.

2 Problem Formulation

In this section, a notion of network topology of UAVs is presented, and a delayed feedback formation control problem of the UAVs is formulated.

The communication topology among UAVs is described by a directed graph. The index set of L followers is defined as $\mathcal{L} = \{1, 2, \cdots, L\}$. Let $\mathcal{G} = (\mathcal{L}_0, \varepsilon)$ denote a directed graph, where $\mathcal{L}_0 = \{0, \mathcal{L}\}$ denote an index set of the leader and L followers, and $\varepsilon \subseteq \mathcal{L}_0 \times \mathcal{L}_0$ is an edge set of paired UAVs. The pairs of UAVs in the directed graph $\mathcal{G}$ are ordered. A directed path is a sequence of ordered edges (i, j), where $i, j \in \mathcal{L}_0$.

Suppose that the follower j ($j \in \mathcal{L}$) only can receive the position information sent from the leader. That is, the positions of the followers only depend on the leader's position. Define an adjacency matrix as $A_c = [c_{ij}]$, where

$$c_{ij} = \begin{cases} 1, & i = 0,\ j \in \mathcal{L} \\ 0, & \text{others} \end{cases} \tag{1}$$

Denote the position and velocity of agent i by $p_i(t)$ and $v_i(t)$, respectively, $i = 0, 1, 2, \cdots, L$. Then one gets

$$\dot{p}_i(t) = v_i(t),\ i = 0, 1, 2, \cdots, L \tag{2}$$

For simplification purpose, the position and velocity information of each agent are further decomposed into three axes as x, y, and z as

$$p_i = \left[p_{ix}\ p_{iy}\ p_{iz} \right]^T,\ v_i = \left[v_{ix}\ v_{iy}\ v_{iz} \right]^T,\quad i = 0, 1, 2, \cdots, L \tag{3}$$

where p_0 and v_0 are predefined position and velocity of the leader.

Define

$$\hat{p}_i(t) = \hat{p}_0(t) - s_i(t),\ i \in \mathcal{L} \tag{4}$$

where $\hat{p}_i = [\hat{p}_{ix}\quad \hat{p}_{iy}\quad \hat{p}_{iz}]^T$ is the expected position of the follower $i(i = 1, 2, \cdots, L)$, and $s_i = [s_{ix}\quad s_{iy}\quad s_{iz}]^T$ represents the relative distance between leader and follower i during the formation process, $i \in \mathcal{L}$.

By Newton's second law, one yields the motion equation of the follower i as

$$f_i(t) = m a_i(t) + k v_i(t) + \vartheta,\quad i \in \mathcal{L} \tag{5}$$

where $f_i = [f_{ix}\ \ f_{iy}\ \ f_{iz}]^T$, $a_i = \dot{v}_i$, k is the air damping coefficient, $\vartheta = [0\ \ 0\ \ mg]^T$ with m the mass of the UAV and g the acceleration of gravity.

Denote

$$\begin{cases} x_i = \begin{bmatrix} x_{i1} & x_{i2} & x_{i3} & x_{i4} & x_{i5} & x_{i6} \end{bmatrix}^T \\ u_i = \begin{bmatrix} f_{ix} & f_{iy} & f_{iz} \end{bmatrix}^T \end{cases} \tag{6}$$

where

$$x_{i1} = p_{ix},\ x_{i2} = v_{ix},\ x_{i3} = p_{iy},\ x_{i4} = v_{iy},\ x_{i5} = p_{iz},\ x_{i6} = v_{iz} \tag{7}$$

Then the state space model of follower i can be expressed as

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Dg,\ i \in \mathcal{L} \tag{8}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{k}{m} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{k}{m} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -\frac{k}{m} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{m} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m} \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \tag{9}$$

Next, for follower i, we define the desired formation trajectory variable as:

$$x_i^s = \begin{bmatrix} x_{i1}^s & x_{i2}^s & x_{i3}^s & x_{i4}^s & x_{i5}^s & x_{i6}^s \end{bmatrix}^T,\ i \in \mathcal{L} \tag{10}$$

where

$$\begin{cases} x_{i1}^s(t) = \hat{p}_{0x}(t) - s_{ix}(t),\ x_{i2}^s(t) = \dot{\hat{p}}_{0x}(t) - \dot{s}_{ix}(t) \\ x_{i3}^s(t) = \hat{p}_{0y}(t) - s_{iy}(t),\ x_{i4}^s(t) = \dot{\hat{p}}_{0y}(t) - \dot{s}_{iy}(t) \\ x_{i5}^s(t) = \hat{p}_{0z}(t) - s_{iz}(t),\ x_{i6}^s(t) = \dot{\hat{p}}_{0z}(t) - \dot{s}_{iz}(t) \end{cases} \tag{11}$$

Then one gets

$$\dot{x}_i^s(t) = Ax_i^s(t) + Bq_i(t),\ i \in \mathcal{L} \tag{12}$$

where

$$q_i(t) = \begin{bmatrix} m[\ddot{\hat{p}}_{0x}(t) - \ddot{s}_{ix}(t)] + kx_{i2}^s(t) \\ m[\ddot{\hat{p}}_{0y}(t) - \ddot{s}_{iy}(t)] + kx_{i4}^s(t) \\ m[\ddot{\hat{p}}_{0z}(t) - \ddot{s}_{iz}(t)] + kx_{i6}^s(t) \end{bmatrix} \tag{13}$$

Define formation error vector as

$$e_i(t) = x_i^s(t) - x_i(t),\ i \in \mathcal{L} \tag{14}$$

and design a formation controller as

$$u_i(t) = u_{if}(t) + u_{ib}(t) \tag{15}$$

where u_{if} and u_{ib} are the feedforward and feedback control laws, respectively.

In this paper, we intend to design the delay-based feedback formation controller (15) for the UAVs system (8) such that the formation error (14) satisfies:

$$\lim_{t \to \infty} e_i(t) = 0,\ i \in \mathcal{L} \tag{16}$$

To obtain the main results, the following Lemma is required.

Lemma 1. *[19] Let ζ be a differentiable function: $[\tau_1, \tau_2] \rightarrow R^n$, and $\tau = \tau_2 - \tau_1$. For any symmetric constant matrix $Z \in R^{n\times n} > 0$, and matrices $P_1 = [M_1 \quad M_2 \quad M_3]$ and $P_2 = [N_1 \quad N_2 \quad N_3]$ with $M_i, N_i \in R^{n\times n}, i = 1, 2, 3$, the following inequality holds:*

$$-\int_{\tau_1}^{\tau_2} \dot{\zeta}^T(s)Z\dot{\zeta}(s)ds \leq \eta^T(t)\left(\Omega + \tau P_1^T Z^{-1} P_1 + \frac{\tau}{3} P_2^T Z^{-1} P_2\right)\eta(t) \tag{17}$$

where

$$\eta(t) = \left[\zeta^T(\tau_2) \;\; \zeta^T(\tau_1) \;\; \frac{1}{\tau}\int_{\tau_1}^{\tau_2} \zeta^T(s)ds\right]^T, \; \Omega = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ * & \phi_{22} & \phi_{23} \\ * & * & \phi_{33} \end{bmatrix} \tag{18}$$

and

$$\begin{cases} \phi_{11} = M_1 + M_1^T + N_1 + N_1^T, & \phi_{12} = -M_1^T + M_2 + N_1^T + N_2 \\ \phi_{13} = M_3 + N_3 - 2N_1^T, & \phi_{22} = -M_2 - M_2^T + N_2 + N_2^T \\ \phi_{23} = -M_3 - 2N_2^T + N_3, & \phi_{33} = -2N_3 - 2N_3^T \end{cases} \tag{19}$$

3 Design of the Formation Controller

In this section, the feedforward and feedback control components $u_{if}(t)$ and $u_{ib}(t)$ in (15) are designed, respectively. Specifically, the existence conditions of the feedback control component are derived.

To compensate the leader-related signal q_i and effects of gravity acceleration, design the feedforward controller as

$$u_{if}(t) = q_i(t) + Hg, \;\; i \in \mathcal{L} \tag{20}$$

where $H = [0 \;\; 0 \;\; m]^T$.

Remark 1. Note that the leader-related signal q_i (13) includes accelerations of the leader and desired velocities and accelerations of followers, which are generalally determined by the topology of the formation problem. Therefore, the feedforward control component can be designed as (20) to compensate the dynamic offset between the leader and the followers thereby enhancing the formation performance of the multiple UAVs system (8).

From (8), (12), and (14) with (15) and (20), one gets

$$\dot{e}_i(t) = Ae_i(t) - Bu_{ib}(t), \;\; i \in \mathcal{L} \tag{21}$$

Design the feedback controller as

$$u_{ib}(t) = K_i e_i(t - d), \;\; i \in \mathcal{L} \tag{22}$$

where K_i is a 3×6 gain matrix of feedback controller to be designed, $d \geq 0$ is an artificial time-delay introduced.

Substituting (22) into (21) yields the closed-loop formation error system as

$$\dot{e}_i(t) = Ae_i(t) - BK_ie_i(t-d),\ i \in \mathcal{L} \tag{23}$$

The following Proposition provides the sufficient conditions of the asymptotical stability of closed-loop formation error system (23).

Proposition 1. *For given scalar $d \geq 0$, the formation error system (23) is asymptotical stable if there exist 6×6 matrices $X > 0$, $Y > 0$, $Z > 0$, $S >$, M_j, N_j, $j = 1, 2, 3$, and 3×6 matrices $K_i, i = 1, 2, \cdots, L$ such that*

$$\begin{bmatrix} \Lambda & \phi_{12} - XBK_i & \phi_{13} & A^TS & \sqrt{d}M_1^T & \sqrt{d}N_1^T \\ * & \phi_{22} - Y & \phi_{23} & -K_i^TB^TS & \sqrt{d}M_2^T & \sqrt{d}N_2^T \\ * & & \phi_{33} & 0 & \sqrt{d}M_3^T & \sqrt{d}N_3^T \\ * & & * & dZ - 2S & 0 & 0 \\ * & & * & * & -Z & 0 \\ * & & * & * & * & -3Z \end{bmatrix} < 0 \tag{24}$$

where $\Lambda = \phi_{11} + XA + A^TX + Y$.

Proof. Construct a Lyapunov-Krasovskii candidate functional as

$$\begin{aligned} V(e_i(t)) =& e_i^T(t)Xe_i(t) + \int_{t-d}^{t} e_i^T(s)Ye_i(s)ds \\ &+ \int_{-d}^{0} ds \int_{t+s}^{t} \dot{e}_i^T(\theta)Z\dot{e}_i(\theta)d\theta \end{aligned} \tag{25}$$

Taking the derivative of $V(e_i(t))$ with respect to t along the trajectory of (23) yields

$$\begin{aligned} \dot{V}(e_i(t)) =& e_i^T(t)\left(XA + A^TX + Y\right)e_i(t) - 2e_i^T(t)XBK_ie_i(t-d) \\ &- e_i^T(t-d)Ye_i(t-d) + d\dot{e}_i^T(t)Z\dot{e}_i(t) - \int_{t-d}^{t} e_i^T(s)Ze_i(s)ds \end{aligned} \tag{26}$$

Note that for any matrix $S > 0$, the following is true:

$$2\left[Ae_i(t) - BK_ie_i(t-d) - \dot{e}_i(t)\right]^T S\dot{e}_i(t) = 0 \tag{27}$$

Let

$$\alpha(t) = \left[e_i(t)\ e_i(t-d)\ \tfrac{1}{d}\int_{t-d}^{t} e_i(s)ds\ \dot{e}_i(t)\right]^T \tag{28}$$

Then, from (26) and (27), and by Lemma 1, one gets

$$\dot{V}(e_i(t)) = \alpha^T(t)[\chi + d\Pi_1^TZ^{-1}\Pi_1 + \frac{d}{3}\Pi_2^TZ^{-1}\Pi_2]\alpha(t) \tag{29}$$

where

$$\Pi_1 = \left[M_1\ M_2\ M_3\ 0\right]^T,\quad \Pi_2 = \left[N_1\ N_2\ N_3\ 0\right]^T \tag{30}$$

and

$$\chi = \begin{bmatrix} \Lambda & \phi_{12} - XBK_i & \phi_{13} & A^T S \\ * & \phi_{22} - Y & \phi_{23} & -K_i^T B^T S \\ * & * & \phi_{33} & 0 \\ * & * & * & dZ - 2S \end{bmatrix} \tag{31}$$

To guarantee the asymptotic stability of the error system (23), the following inequality is needed:

$$\chi + d\Pi_1^T Z^{-1} \Pi_1 + \frac{d}{3} \Pi_2^T Z^{-1} \Pi_2 < 0 \tag{32}$$

which is equivalent to the one in (24) by Schur complements. This completes the proof.

To solve the gain matrix K_i in (22), multiply the left-hand side of the inequality (24) by diag$\{X^{-1}, X^{-1}, X^{-1}, S^{-1}, X^{-1}, X^{-1}\}$ and its transpose, respectively, and denote $\bar{X} = X^{-1}$, $\bar{Y} = X^{-1}YX^{-1}$, $\bar{Z} = X^{-1}ZX^{-1}$, $\bar{S} = S^{-1}$, $\widetilde{Z} = S^{-1}ZS^{-1}$, $\bar{M}_j = X^{-1}M_jX^{-1}, \bar{N}_j = X^{-1}N_jX^{-1}, j = 1, 2, 3$. Then one yields

$$\begin{bmatrix} \bar{\Lambda} & \bar{\phi}_{12} - B\bar{K}_i & \bar{\phi}_{13} & \bar{X}A^T & \sqrt{d}\bar{M}_1^T & \sqrt{d}\bar{N}_1^T \\ * & \bar{\phi}_{22} - \bar{Y} & \bar{\phi}_{23} & -\bar{K}_i^T B^T & \sqrt{d}\bar{M}_2^T & \sqrt{d}\bar{N}_2^T \\ * & * & \bar{\phi}_{33} & 0 & \sqrt{d}\bar{M}_3^T & \sqrt{d}\bar{N}_3^T \\ * & * & * & d\widetilde{Z} - 2\bar{S} & 0 & 0 \\ * & * & * & * & -\bar{Z} & 0 \\ * & * & * & * & * & -3\bar{Z} \end{bmatrix} < 0 \tag{33}$$

where $\bar{\Lambda} = \bar{\phi}_{11} + A\bar{X} + \bar{X}A^T + \bar{Y}$, and

$$\begin{cases} \bar{\phi}_{11} = \bar{M}_1 + \bar{M}_1^T + \bar{N}_1 + \bar{N}_1^T, & \bar{\phi}_{12} = -\bar{M}_1^T + \bar{M}_2 + \bar{N}_1^T + \bar{N}_2 \\ \bar{\phi}_{13} = \bar{M}_3 + \bar{N}_3 - 2\bar{N}_1^T, & \bar{\phi}_{22} = -\bar{M}_2 - \bar{M}_2^T + \bar{N}_2 + \bar{N}_2^T \\ \bar{\phi}_{23} = -\bar{M}_3 - 2\bar{N}_2^T + \bar{N}_3, & \bar{\phi}_{33} = -2\bar{N}_3 - 2\bar{N}_3^T \end{cases} \tag{34}$$

Based on above analysis, we have following Proposition.

Proposition 2. *For given scalar $d \geq 0$, if there exist 6×6 matrices $\bar{X} > 0$, $\bar{Y} > 0$, $\bar{Z} > 0$, $\widetilde{Z} > 0$, $\bar{S} > 0$, $\bar{M}_j$, $\bar{N}_j$, $j = 1, 2, 3$, and 3×6 matrices $\bar{K}_i, i = 1, 2 \cdots, L$ such that the inequality (33) holds, then the gain matrices $K_i, i = 1, 2, \cdots, L$ of the delayed feedback controller (22) are solvable, and*

$$K_i = \bar{K}_i \bar{X}^{-1} \tag{35}$$

Remark 2. Proposition 2 provides a method to solve gain matrix K_i of feedback controller (22). In fact, for a given time-delay d artificially introduced, if the inequality (33) is feasible, then the gain matrix K_i can be computed. Further, combining with (20) and (22), the delayed feedback formation controller (15) can be obtained.

Remark 3. Based on the linear matrix inequality (33), the maximum admissible time-delay $d_{\max}$ intentionally introduced can be computed for the multiple UAVs system (8), and the effects of different time-delay d on the formation performance of the multiple UAVs system are different, which will be disscussed below.

4 Simulation Examples

In this section, two examples regarding two different formation patterns are presented to show the effectiveness of the proposed formation control scheme for a multiple UAVs system. Then the effects of the time-delays introduced on the formation performance of the system are discussed.

4.1 Parameters of the UAVs System and Formation Patterns

In (8), suppose that there are eight followers, i.e., $L = 8$. The mass m of each UAV is 5kg, and the air dumping coefficient k is 3N·s/m. The desired flight path $\hat{p}_0(t)$ of the leader is given by

$$\hat{p}_0(t) = [10t \;\; 10\sin(0.1t) \;\; 100(1 - e^{-0.1t})]^T, \; t \geq 0 \tag{36}$$

In the two cases of formation pattern, i.e., 1-shape and V-shape, the initial states and the desired offsets are listed as follows:
Case I. 1-shape formation pattern

$$\begin{aligned} x_i(0) &= \begin{cases} \begin{bmatrix} 0 \; 0 \; -10i \; 0 \; 0 \; 0 \end{bmatrix}^T, i = 1,2,3,4 \\ \begin{bmatrix} 0 \; 0 \; 10(i-4) \; 0 \; 0 \; 0 \end{bmatrix}^T, i = 5,6,7,8 \end{cases} \\ s_i(t) &= \begin{cases} \begin{bmatrix} 0 \; 10i \; 0 \end{bmatrix}^T, i = 1,2,3,4 \\ \begin{bmatrix} 0 \; -10(i-4) \; 0 \end{bmatrix}^T, i = 5,6,7,8 \end{cases} \end{aligned} \tag{37}$$

Case 2. V-shape formation pattern

$$\begin{aligned} x_i(0) &= \begin{cases} \begin{bmatrix} -80i \; 0 \; -20i \; 0 \; 0 \; 0 \end{bmatrix}^T, i = 1,2,3,4 \\ \begin{bmatrix} -80(i-4) \; 0 \; 20(i-4) \; 0 \; 0 \; 0 \end{bmatrix}^T, i = 5,6,7,8 \end{cases} \\ s_i(t) &= \begin{cases} (i(5-\cos(0.1t)) - 2\sin(0.1t)) \begin{bmatrix} 8 \; 3 \; 0 \end{bmatrix}^T, i = 1,2,3,4 \\ (i(5-\cos(0.1t)) - 2\sin(0.1t)) \begin{bmatrix} 8 \; -3 \; 0 \end{bmatrix}^T, i = 5,6,7,8 \end{cases} \end{aligned} \tag{38}$$

To investigate the performance of the multiple UAVs system under designed formation controller, we introduce two performance indices with respect to formation error and control cost as:

$$J_{ei} = \int_0^\infty e_i^T(t) e_i(t) dt, \quad J_{ui} = \int_0^\infty u_i^T(t) u_i(t) dt \tag{39}$$

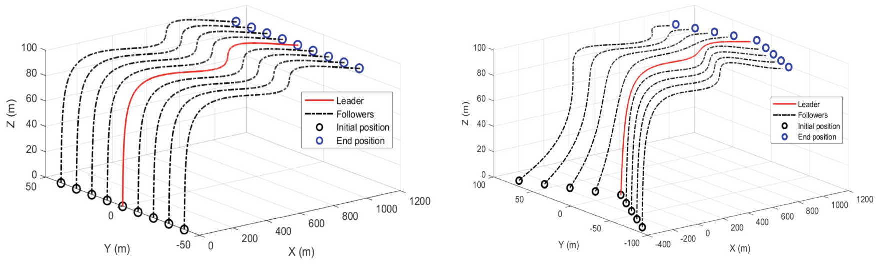
In what follows, in the aforesaid two cases, a delayed feedback formation controller with feedforward components is designed, and the performances of the formation control systems and the effects of different time-delay on the formation control are discussed.

4.2 Formation Control Effects of the UAVs System

Based on (13) and (37) (or (38)), the feedforward component $u_{if}(t)$ in (20) can be determined. To design the feedback control component $u_{ib}(t)$, set $d = 0.35\,\mathrm{s}$. Then by Proposition 2, the gain matrix K_i is computed as

$$K_i = \begin{bmatrix} 2.4566 & 5.7279 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.4566 & 5.7279 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.4566 & 5.7279 \end{bmatrix}, \ i = 1, 2, \cdots, 8 \tag{40}$$

Further the delay-based feedback formation controllers (DFFCs) in the form (15) can be obtained. The two formation controllers are denoted as DFFC1 for case 1 and DFFC2 for case 2, respectively. As the DFFC1 and DFFC2 are applied to the UAVs system, the formation control results are depicted in Fig. 1(a) for case I and Fig. 1(b) for case II, respectively. The figures show that under the designed formation controllers, the followers can track the leader effectively. In addition, the 1-shape formation pattern (case I) and V-shape formation pattern (case II) can be realised for all agents in the UAVs system.



(a) Formation control under DFFC1. (b) Formation control under DFFC2.

Fig. 1. Formation control result of UAVs, $d = 0.35$ s.

4.3 Effects of Time-Delay on Formation Performance

By Proposition 2, it can be computed that the maximum admissible time-delay intentionally introduced is about 1.33s. To analyze the effects of the time-delay on the formation performance and control cost by the multiple UAVs system, let the value of time-delay d increase from 0s with a step 0.01s. Then under the DFFC1 and DFFC2 designed in Subsect. 4.2, one yields the performance indices (39) regarding formation error and control cost of UAVs, which are listed in Table 1 for case I and Table 2 for case II, respectively.

It is observed from Tables 1 and 2 that with the increase of time-delay d, the whole formation error of the multiple UAVs system and the control cost become large gradually. Specifically, if $d = 0$ s, the formation error and control cost are the smallest, while if $d = 1.33$ s, the former error and control cost are the largest,

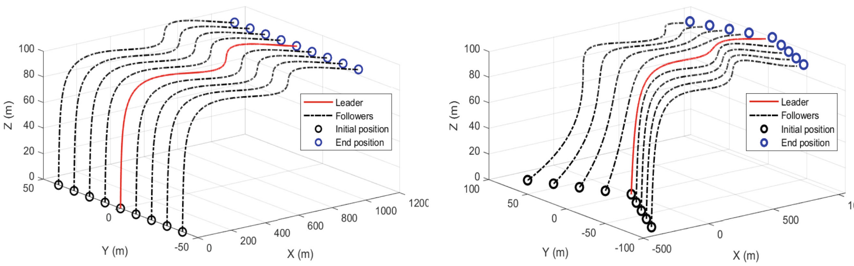
Table 1. Performance indices of UAVs with DFFC1 for different time-delays.

d (s)	0	0.08	0.15	0.25	0.35	0.70	1.00	1.33
$J_e(10^3)$	1.4357	1.5152	1.5914	1.7134	1.8543	2.5959	3.9574	9.5016
$J_u(10^6)$	2.9689	2.9708	2.9727	2.9756	2.9789	2.9956	3.0248	3.1395

Table 2. Performance indices of UAVs with DFFC2 for different time-delays.

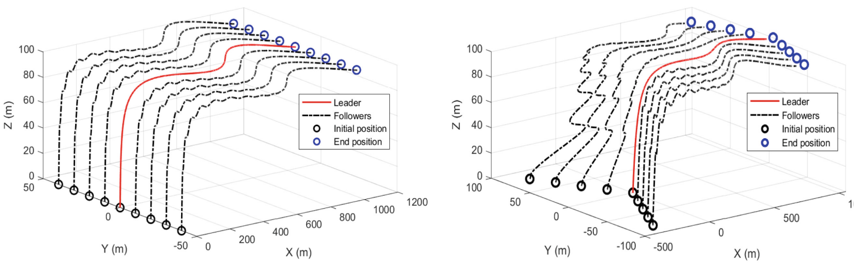
d (s)	0	0.08	0.15	0.25	0.35	0.70	1.00	1.33
$J_e(10^5)$	3.3508	3.3928	3.4313	3.4900	3.5537	3.8406	4.2678	5.7346
$J_u(10^6)$	3.6681	3.7144	3.7583	3.8276	3.9066	4.3085	5.0181	7.8303

which can be found from Figs. 2(a) and 2(b) for $d = 0$ s and Figs. 3(a) and 3(b) for $d = 1.33$ s, respectively. In fact, if the value of introduced time-delay d is larger than 1 s, the relatively larger chattering phenomenon occurs. Consequently, the formation performance of the UAVs under the designed formation controller degrades gradually. Therefore, it is significant to choose a proper delay used for the formation controller design for the multiple UAVs system.



(a) Formation control under DFFC1. (b) Formation control under DFFC2.

Fig. 2. Formation control result of UAVs, $d = 0$ s.



(a) Formation control under DFFC1. (b) Formation control under DFFC2.

Fig. 3. Formation control result of UAVs, $d = 1.33$ s.

5 Conclusion

In this paper, by introducing time-delay intentionally, a delay-based feedback formation control scheme with feedforward components has been developed. Based on leader-follower formation mode, a moving equation of UAVs has been established in a three-dimensional space. A delay-based feedback formation controller with feedforward components has been designed for the multiple UAVs system. By using Krasovskii stability theory, the existence and design method of the delayed formation controller have been obtained. Simulation results have been provided to demonstrate the effectiveness of the proposed formation control scheme for the multiple UAVs system. The results show that when the time-delay is 0.35s, the system performance is relatively ideal, and when it is greater than 1.33s, the system trembles severely.

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